DISEQUILIBRIUM ESTIMATION OF SINGLE MARKET EFFECTIVE DEMAND THEORY

EDWARD OCKOWSKI*

Charles Sturt University—Riverina—Australia

This paper reconciles ‘markets in disequilibrium’ econometric methods with single market effective demand theory. Effective demand is formulated assuming agent asymmetry; some agents manipulate the deterministic rationing scheme, others do not. This theory reconciles the concept of deterministic manipulability with actual rationing. An econometric model broadly based on this theoretical construct is outlined using the alternative GTZ and MN disequilibrium error specifications and various expectations assumptions. The GTZ specification with lagged expectations seems the most plausible. Monte Carlo simulations show that these more complex effective demand specifications are computationally feasible and that the effects of incorrectly specifying a notional model may be disastrous.

1. INTRODUCTION

Methods for estimating single ‘markets in disequilibrium’ are now commonplace, Quandt’s (1988) comprehensive survey of this literature illustrates an abundance of estimation methods, models and applications. In particular, demand and supply equations, a quantity transaction rule and a price adjustment equation are specified, and methods, to estimate and apply such constructs are described. Literature on the theoretical economic aspects of single ‘markets in disequilibrium’ also exists, see Grossman (1974), McCafferty

*Research for this paper was undertaken while at La Trobe University. I wish to thank Godfrey Lubulwa and an anonymous referee for their comments.
(1977), Benassy (1982, Chapters 1-3) and Weinrich (1984). Discussions formulise agent optimising behaviour in single isolated disequilibrium trading markets by defining effective demand. Rationing aspects of disequilibrium trading are highlighted and thus notions such as, manipulability of rationing schemes, perceptions of rationing and transaction costs associated with expressing demands are discussed. Even though some overlapping between the two literature streams exists (i.e., both typically use the minimum condition to describe trading) in general, the econometric literature ignores the concept of effective demand. This paper rectifies this discrepancy by providing estimation procedures for single market effective demand theory.

The next section outlines and dismisses the econometric justification for specifying notional demand and supply functions in disequilibrium models. In response a new effective demand theory is outlined based on asymmetric information, resources and manipulability; deterministic manipulability is reconciled with actual rationing. Section 3 presents the general effective demand econometric model and proposes two alternative stochastic error specifications. Agent expectations of rationing play the crucial role in such models and so section 4 discusses maximum likelihood estimation methods for different expectation assumptions. Section 5 examines the computational feasibility of the effective demand model using Monte Carlo simulations and Section 6 concludes.

2. SINGLE MARKET EFFECTIVE DEMAND THEORY

Most empirical single market disequilibrium studies specify demand and supply functions based on conventional 'choice theoretic' notional optimisation programs. Implicitly, ex-post rationing is assumed, that is, agents expect to gain notional desires at each trading session and hence express notional demands, but once at the market, rationing takes place (ex-post) and the minimum condition describes the rationing occurrence; see for example, Lafont and Garica (1977, p. 1189). In contrast, ex-ante rationing describes effective demand theory. Here agents realise that market clearing is unlikely and hence form an expectation of rationing (ex-ante), consequently adjusting demands from notional to effective amounts. Given the disequilibrium trading assum-

1. In the context of single markets, there exists one notable exception where theory and estimation have been reconciled, see Eaton and Quandt (1983). This paper abstracts from multi-market disequilibrium models where, in contrast to single market models, the sole motivation for econometric models like Gourieroux, Lafont and Monfort (1980) rests with theoretical models like Barro and Grossman (1971). This abstraction is possible, given the general spillover effect definition of effective demand in multi-markets, which disappears, when only one market exists in isolation.
DISEQUILIBRIUM ESTIMATION

Consider the following review of some of the existing effective demand literature. Typically, all agents on the long-side of a disequilibrium trading market will not be able to attain all their notional desires. Consequently, this limited amount of supply must be allocated to individual agents by a rationing scheme. In general, Benassy (1982, Chaps. 1-3), describes two types of rationing schemes; non-manipulable and manipulable. Under non-manipulability an agent cannot gain more than the allocated rationed amount. Conversely, a manipulable rationing scheme is one which as a result of expressing a higher demand (and expending more resources) one can gain or 'manipulate' a higher transaction.

A further aspect of the rationing nature of trading concerns whether the allocation mechanism is deterministic or stochastic. The stochastic rationing scheme has been studied by Green (1980) and Weinrich (1984). These authors argue that one must specify a stochastic rationing scheme to meet the two basic requirements that effective demand theories should possess, i.e., (a) effective demand stems from optimisation principles, (b) the difference between expected trade and effective demand is based on dissatisfaction. The first requirement is indisputable, the second is needed to justify any price adjustment theory based on excess demand.

Distinct from these crucial rationing aspects of disequilibrium markets, the role of transaction costs associated with expressing demands is also seen as important. Clearly, when manipulation of a rationing scheme occurs it is not without cost. For example, one may manipulate a queuing scheme by expending resources to arrive at the queue earlier. These costs will limit the unacceptable overbidding demand behaviour typically associated with cost-free manipulable rationing schemes (see Benassy 1982, pp. 34-36). Studies with transaction costs include; McCafferty (1977), who models the costs in searching for opened stores and Eaton and Quandt (1983) who point to the costs associated with attempts to find jobs in the labour market.

Given this brief review now consider an alternative effective demand theory. Even though preliminary in nature, these new notions capture some previously untapped and important aspects of actual disequilibrium trading. In what follows we emphasise the asymmetric nature of manipulable rationing schemes. To set the scene consider the following example.

Assume the rationing scheme is queuing. An agent might be able to manipulate this scheme by expending resources by arriving early or by placing

---

2. The general point that conventional/notional functions are not relevant in disequilibrium situations has been long recognised (Hicks 1939, p. 128) and is widely accepted (Hendry 1982, pp. 65-66).
other representatives in the queue. Here manipulation is with transactions
costs. However, given the minimum condition and hence the fixed available
supply, it is clear that not all agents can manipulate the scheme in the same
way. Manipulation by one agent is necessarily at the expense of another
agent. Agent asymmetry exists, some agents manipulate the scheme others do
not.

Clearly, only a subset of agents can at any one time manipulate a scheme, some
agents either have more resources or more information and hence are capable
of manipulating the scheme. This asymmetry is important and does reconcile
the notion of manipulability with actual rationing. On the other hand, it is
the naive assumption that all agents can manipulate a deterministic rationing
scheme that leads to the nonsensical overbidding manipulable result, here
manipulability is at variance with actual rationing.

We shall now formulate these ideas. In doing so it will become apparent that
we can stay in the deterministic realm and satisfy the two idealistic requirements
of an effective demand theory. Assume we have *n* market agents each agent
has an identical notional demand \( q^d \) (aggregation provides \( Q^d = nq^d \)), each
agent forms the same expectation of total aggregate supply \( Q^{es} \) and each agent
knows that there exist \( n \) agents in total. Introduce agent asymmetry by assum-
ing that \( n \) can be divided into two groups, \( n_1 \) and \( n_2 \) (\( n_1 + n_2 = n \)). Group 1
(size \( n_1 \)) lacks resources and information and hence does not try to mani-
pulate the scheme, these agents act as non-manipulators. Group 2 (size \( n_2 \)) has
resources and information and is capable of manipulating the scheme. It is
clear that \( n_2 \) must be small relative to \( n_1 \) for us to reconcile manipulability
with actual rationing, the precise size requirements will be detailed later.

Let us describe the optimising behaviour which leads to effective demand.
Assume there exist transaction costs for all demand expression, given these
costs agents in group one will express demands equal to their expected con-
straints. Clearly, if they expect the scheme to be non-manipulable then any
demand expression beyond the constraint incurs cost at no extra benefit.
Given the assumption of identical agents then each agent in group one expres-
ses the following effective demand:

\[
\hat{q}_i = \frac{Q^{es}}{n}
\]  

(1)

To model manipulability assume that the agents of group two know the true
rationing scheme is a proportional one. Proportional rationing schemes are
typical of bonds and equity markets, see Benassy (1975: 505). Here by
expressing higher demands (and greater resources) agents in group two do
manipulate the scheme. The relation between each agent two’s demand \( q^2 \)
and transaction \( q_s \) can be written as:

\[
q_s = Q^{se} \ast (\tilde{Q}_d/Q_d^d)
\]

(2)

Here \( \tilde{Q}_d \) defines total aggregate effective demand.9

Assume that each agent in group two can manipulate the scheme to gain their notional desires \( \tilde{q}_s \), that is, \( n_1, n_2, \tilde{q}_d \) and \( Q^{se} \) are of appropriate size. Therefore, to determine effective demand for group two, equate (2) to \( \tilde{q}_d \). To make the solution for \( \tilde{q}_d \) tractable assume that all agents in group two manipulate in exactly the same way and each agent in group two knows that the others are doing so. Also assume that group two agents know that equation (1) represents effective demand for group one. Under these circumstances (2) becomes:

\[
\tilde{q}_d = q_s = Q^{se} \ast [(\tilde{q}_d^i/(n_1/n)Q^{se} + n_2\tilde{q}_d^2)]
\]

(3)

Equation (3) can be solved for a unique effective demand for each identical agent in group two:

\[
\tilde{q}_d = (Q^dQ^{se}(n_1/n))/(\tilde{Q}_d - \tilde{q}_d)
\]

(4)

The requirements on the group sizes and expected supply, . . . etc., for (3) to hold can be described. For \( \tilde{q}_d \) to be finite, that is, all demanders in group two capable of gaining their notional desires, we need .4

\[
(n_1/n_2) > (\tilde{Q}_d - Q^{se})/Q^{se}
\]

(5)

That is, the ratio of non-manipulators to manipulators must be greater than the proportion of expected excess demand. For example, if excess demand is of 100% in magnitude (i.e., \( \tilde{Q}_d \) is double \( Q^{se} \)) then \( n_1 > n_2 \), that is there must be more non-manipulators than manipulators. The requirement of (5) does not seem prohibitive.

3. For convenience the notation is gathered as : \( Q, \tilde{Q}_d, \tilde{q}_d \) as aggregate transaction, notional demand, and effective demand, respectively; \( q, q^i, q_d^i (i = 1, 2) \) as each agent's (in group one or two) individual transaction, notional demand, and effective demand, respectively; \( Q, Q_d^i, Q_d^d (i = 1, 2) \) as the aggregate within group (one or two) transaction, notional demand, and effective demand.

4. Equation (5) is derived as follows. For \( \tilde{q}_d \) to be finite \( Q^{se} - \tilde{Q}_d > 0 \), however by definition \( \tilde{Q}_d = (n_2/n) \tilde{Q}_d \). Equation (5) then automatically follows from \( n_1 + n_2 = n \).
To determine aggregate effective demand, sum over all $n_1$ identical agents in group one using (1) and overall $n_2$ identical agents in group two using (4). In total effective demand is:

$$\bar{Q}^d = \frac{(Q^{se})^2}{(n/n_1)} (Q^{se} - \bar{Q}_2)$$  \hspace{1cm} (6)

To make matters more lucid (using $(n_2/n)\bar{Q}^d = \bar{Q}_2^d$) equation (6) can be written as:

$$\bar{Q}^d = \bar{Q}^d + n(\bar{Q}^d - Q^{se})$$  \hspace{1cm} (7)

with

$$n = \frac{(n_2\bar{Q}^d - n_1Q^{se})}{n(Q^{se} - \bar{Q}_2)}$$  \hspace{1cm} (8)

Equation (7) describes effective demand as departures from national demand, these departures depend upon some value $n$ and the total amount of expected rationing $(\bar{Q}^d - Q^{se})$.

It will prove useful to investigate the properties of $\alpha$. It is easy to show that $\alpha > -1$, otherwise $(\bar{Q}^d < Q^{se})$ which is non-sensical. The sign of $\alpha$ depends upon the relation between $(n_2/n_1)$ and $(Q^{se}/\bar{Q}^d)$; if $(n_2/n_1) > (Q^{se}/\bar{Q}^d)$ then $\alpha > 0$, negative otherwise. Clearly, $(Q^{se}/\bar{Q}^d) < 1$ and thus it can be suggested that $n_2 > n_1$ always produces $\alpha > 0$, while $\alpha < 0$ can only arise if $n_2 < n_1$. In a general descriptive sense this implies that if many manipulators exist then effective demand will be greater than notional demand, while an effective demand less than notional demand can only arise if we have only a few manipulators. In other words, the harsher the rationing the greater the effective demand.

In Section 3 when discussing estimation, equation (7) will be employed, thus it will be necessary to investigate the constancy or otherwise of $\alpha$. It can be shown that the elasticity of $\alpha$ to $Q^{se}$ is equal to that to $\bar{Q}^d$ but with opposite sign, that is, a 10% increase in both $Q^{se}$ and $\bar{Q}^d$ will leave $\alpha$ unaltered. In other words, even though $(\bar{Q}^d - Q^{se})$ will vary, $\alpha$ is constant if $[Q^d - Q^{se}/\bar{Q}^d]$, the proportion of rationing, remains constant.

It is clear that (7) is based on explicit optimising behaviour, that is, group one agents act as optimisers by not expressing demands beyond their expected constraints, while group two agents optimise by manipulating the scheme to gain their notional desires. However, does the difference between trade and demand in (7) provide a measure of dissatisfaction? To investigate this rewrite (7) as:

$$(\bar{Q}^d - Q^{se}) = (1 + n)(\bar{Q}^d - Q^{se})$$  \hspace{1cm} (9)
The LHS of (9) measures the difference between demand and expected trade, since \( a > -1 \) then it is always positive. On the RHS, \((\bar{Q}^d - Q^{es})\) provides a measure of dissatisfaction. Consider the following argument. All group two agents are satisfied \( \bar{Q}_e = \bar{Q}^{d^2} \), the transactions received by group one are those left behind by the manipulators \((\bar{Q}^{es} - \bar{Q}^{d^1})\). Thus the difference between total desires \((\bar{Q}^d + \bar{Q}^{d^2})\) and total trades \([(\bar{Q}^{es} - \bar{Q}^{d^1}) + \bar{Q}^{d^2}]\) is \((\bar{Q}^d - Q^{es})\). Thus the difference between demand and trade is explicitly dependent upon the amount of dissatisfaction. The greater the dissatisfaction the greater the difference. Effectively, group one’s dissatisfaction is not expressed directly through the demand trade difference but indirectly through group two’s manipulability. Given harsher expected rationing, group two must manipulate the scheme to a greater extent to gain their notional desires. This increases the gap between demand and trade. Given the greater group two manipulability less is left for group one and hence becomes more dissatisfied.

Given this setting, we are now in a position to provide an econometric representation of single market effective demand theory. The next section provides the general econometric model for such purposes.

3. THE ECONOMETRIC MODEL

As stressed, the important issue is to model the expectation of rationing by agents explicitly. This is important as agents are seen to determine effective demands based on their expectation of rationing. Assume that demanders wish to make the notional aggregate demand \( \bar{Q}_e^d \) and assume they form an expectation of supply \( Q^{es} \). Similarly assume suppliers desire \( \bar{Q}_s^d \) in aggregate and form the expectation of demand \( Q^{ds} \). Given this notation, four possible cases of expected rationing may eventuate:

\[
\begin{align*}
\bar{Q}_e^d > Q^{es} \quad &\text{and} \quad \bar{Q}_s^d > Q^{ds} \\
\bar{Q}_e^d < Q^{es} \quad &\text{and} \quad \bar{Q}_s^d < Q^{ds} \\
\bar{Q}_e^d > Q^{es} \quad &\text{and} \quad \bar{Q}_s^d < Q^{ds} \\
\bar{Q}_e^d < Q^{es} \quad &\text{and} \quad \bar{Q}_s^d < Q^{ds}
\end{align*}
\]

(10) (11) (12) (13)

In cases (10) and (11) demanders expect to be rationed, while in cases (10) and (12) suppliers expect to be rationed. Both sets of agents expect to be rationed in (10) and unrationed in (13). Cases (10) and (13) are just as plausible as (11) and (12) since expectations may be inaccurate.
In forming the effective demand functions, employ the notions of section two and equation (7). Demanders and suppliers are treated symmetrically. In what follows ‘α’ in (7) is treated as a constant. This is necessarily an approximation which is given some justification in section two. This linear formulation is similar to Grossman’s (1974) formulation of manipulation and transaction costs and similar to effective demands used in multi-market models where a temporal across market spillover effects are modelled. The effective demand model is:

\[ \tilde{Q}^d_t = \bar{Q}^d_t + \alpha \max \left( 0, Q^d_t - Q^e_t \right) \]  

\[ \tilde{Q}^s_t = \bar{Q}^s_t + \beta \min \left( 0, Q^e_t - \bar{Q}^s_t \right) \]  

The notation is similar to that in Section 2. As an illustrative example consider case (11); here \( \tilde{Q}^d_t > Q^e_t \), that is, demanders expect to be rationed, clearly (14) implies that effective demand will differ from notional demand according to the theory of section two and equation (7). For supply, \( \tilde{Q}^s_t < Q^e_t \), suppliers expect to be unrationed hence (15) implies that notional supply will be expressed.

In these formulations restrictions must be placed on \( \alpha \) and \( \beta \). As described in Section 2, assume \( \alpha > -1 \) (similarly \( \beta < 1 \)). Further, given normal sloping demand/supply curves, these assumptions are needed to provide excess demand price adjustment market stability and conventional price change sample separations.

To make the model defined by (14) and (15) statistically operational error terms must be appended. Errors in specification, measurement, aggregation, market operations, . . . etc., might be imposed in two alternative ways. First, assume \( a la \) the GTZ (Ginsburgh, Tishler and Zang (1980)) specification of disequilibrium markets, that the market process itself is constantly shocked and hence add a stochastic error term to the minimum condition. Alternatively, \( a la \) the MN (Maddala and Nelson (1974)) specification, we might claim error in specifying the behavioural functions of market agents.

Ignoring the non-zero error co-variance possibility, these two alternative specifications can be described as special cases of a general stochastic specification of market operations. Add to equations (14) and (15) the following:

\[ \tilde{Q}^d_t = \alpha_0 X^d_t + \alpha_1 P_t + \epsilon^d_t \]  

\[ \tilde{Q}^s_t = \beta_0 X^s_t + \beta_1 P_t + \epsilon^s_t \]  

\[ Q_t = \min \left( \tilde{Q}^d_t, \tilde{Q}^s_t \right) + \epsilon_t \]
DISEQUILIBRIUM ESTIMATION

\[ U_i^d \sim (0, \sigma^d_i), \quad U_i^s \sim N(0, \sigma^2_i), \quad U_i^q \sim N(0, \sigma^2_q) \]

GTZ Specification: \[ U_i^d = U_i^q = 0 \] (19)

MN Specification: \[ U_i^q = 0 \] (20)

In the above, \( X_i^d \) and \( X_i^s \) represent demand and supply exogenous regressors and \( P_t \) price. In the next section we discuss maximum likelihood estimation methods for both specifications and various expectation assumptions concerning \( Q^{de} \) and \( Q^{se} \).

4. MAXIMUM LIKELIHOOD METHODS

Maximum likelihood methods will be discussed for two expectation assumptions; perfect expectations (\( Q^{de} = \bar{Q}_t^d \) and \( Q^{se} = \bar{Q}_t^s \)) and non-perfect expectations (\( Q^{de} \neq \bar{Q}_t^d \) and \( Q^{se} \neq \bar{Q}_t^s \)). In the case of perfect expectations the four cases of expected rationing collapse to two (11) and (12), for now (10) and (13) implying incorrect expectations are logically impossible. In the general case of non-perfect expectations likelihood functions are more complex as all four cases require evaluation.

Unfortunately, the perfect expectation assumption poses both conceptual and estimation problems for both specifications, this is due to the contemporaneous nature of the expectation. Conceptually, perfect expectations implies that agents know what will happen before it actually does! For example, in the MN specification the PDF of \( Q_t \) depends on \( \bar{Q}_t^d \) and \( \bar{Q}_t^s \) through the minimum condition, however in some cases \( \bar{Q}_t^d \) and \( \bar{Q}_t^s \) will explicitly depend on \( Q_t \). How can agents know \( Q_t \) before it is actually determined? An undesirable circular causation exists.

This conceptual problem surfaces in both the GTZ and MN likelihood functions via the absences of \( \alpha \) and \( \beta \), implying that these parameters are not estimable. Further even the addition of a price adjustment equation [i.e. \( \Delta P_t = \lambda(\bar{Q}_t^d - \bar{Q}_t^s) \)] to the MN specification poses problems of parametric identification for \( \lambda, \alpha \) and \( \beta \).

Next consider non-perfect expectations and the MN specification. Unfortunately the likelihood function becomes computationally intractable due to the necessary inclusion of lagged endogenous variables in demand and supply equations. The intractability surfaces through the need to evaluate multi-dimensioned normal integrals, see Lee (1984).

The only feasible specification is the GTZ specification and non-perfect expectations. Under the assumption that \( Q^{de} \neq \bar{Q}_t^d \) and \( Q^{se} \neq \bar{Q}_t^s \) the like-
likelihood function requires the defining of six step functions. First, we need to
determine the sign of \( (\bar{Q}_t^d - \bar{Q}_t^r) \), however, \( \bar{Q}_t^d \) and \( \bar{Q}_t^r \) are dependent on the
signs of \( (\bar{Q}_t^d - Q^{se}) \) and \( (Q^{de} - \bar{Q}_t^r) \). Four combinations can result from the
signs of \( (\bar{Q}_t^d - Q^{se}) \) and \( (Q^{de} - \bar{Q}_t^r) \), hence four step functions need defining
(i.e., \( X_{1t}, X_{2t}, X_{3t}, X_{4t} \)). Further, another two step functions are needed for the
sign determination of \( (\bar{Q}_t^d - Q^{se}) \) and \( (Q^{de} - \bar{Q}_t^r) \).

To write the likelihood function define the following deterministic step functions:

\[
\begin{align*}
[\bar{Q}_t^d - \bar{Q}_t^r (1 - \beta_d) - \beta_s Q^{de}] < 0 & \quad X_{1t} = 0 \\
\text{otherwise} & \quad X_{1t} = 1 \quad (21)
\end{align*}
\]

\[
\begin{align*}
[ (1 + \alpha_d) \bar{Q}_t^d - \beta_s Q^{de} - (1 - \beta_s) \bar{Q}_t^r - \alpha_s Q^{se}] \leq 0 & \quad X_{2t} = 0 \\
\text{otherwise} & \quad X_{2t} = 1 \quad (22)
\end{align*}
\]

\[
\begin{align*}
[\bar{Q}_t^d - \bar{Q}_t^r] < 0 & \quad X_{3t} = 0 \\
\text{otherwise} & \quad X_{3t} = 1 \quad (23)
\end{align*}
\]

\[
\begin{align*}
[ (1 + \alpha_d)\bar{Q}_t^d - \alpha_s Q^{se} - \bar{Q}_t^r] < 0 & \quad X_{4t} = 0 \\
\text{otherwise} & \quad X_{4t} = 1 \quad (24)
\end{align*}
\]

\[
\begin{align*}
[\bar{Q}_t^d - Q^{se}] \leq 0 & \quad Y_t = 0 \\
\text{otherwise} & \quad Y_t = 1 \quad (25)
\end{align*}
\]

\[
\begin{align*}
[Q^{de} - \bar{Q}_t^r] < 0 & \quad Z_t = 0 \\
\text{otherwise} & \quad Z_t = 1 \quad (26)
\end{align*}
\]

Further define the following densities:

\[
\begin{align*}
f_1(Q_t) \quad \text{for} \quad Q_t &= (1 + \alpha_d)\bar{Q}_t^d - \alpha_s Q^{se} + U_t^d \quad (27) \\
f_2(Q_t) \quad \text{for} \quad Q_t &= \bar{Q}_t^d + U_t^d \quad (28) \\
g_1(Q_t) \quad \text{for} \quad Q_t &= (1 - \beta_d)\bar{Q}_t^r + \beta_s Q^{de} + U_t^r \quad (29) \\
g_2(Q_t) \quad \text{for} \quad Q_t &= \bar{Q}_t^r + U_t^r \quad (30)
\end{align*}
\]
Here $\tilde{Q}_t$ and $\tilde{Q}_t^n$ are defined without error. All this notation allows us to write the likelihood function as:

\[
L = \prod_{t=1}^{n} \left[ f_{1}(Q_t) Y_t (1 - Z_t) (1 - X_{st}) + Z_t (1 - X_{at}) \right] \\
+ f_{2}(Q_t) (1 - Y_t) [(1 - Z_t) (1 - X_{st}) + Z_t (1 - X_{at})] \\
+ g_{1}(Q_t) (1 - Z_t) [(1 - Y_t) X_{st} + Y_t X_{at}] \\
+ g_{2}(Q_t) \Phi \left[ (1 - Y_t) X_{st} + Y_t X_{at} \right]
\]

(31)

As described in Goldfeld and Quandt (1972, Chap. 9) and Tishler and Zang (1979), the six step functions must be replaced by some continuous approximation to enable conventional gradient optimisation. Either the Goldfeld and Quandt cumulative normal integral, or the Tishler and Zang spline approximations can be applied.

This completes the discussion of likelihood functions. Clearly, the assumption of perfect expectations faces both conceptual and statistical problems, further it assumes ‘perfect’ information, thus in general it is not recommended. The MN specification with non-perfect expectations is computationally intractable and hence not operational. Only the GTZ specification with non-perfect expectations remains viable. To investigate the computational feasibility of this six step function method Monte Carlo simulations are employed in the next section.

5. MONTE CARLO SIMULATIONS

In this section two issues are investigated. First, is the GTZ specification with non-perfect expectations computationally feasible? This issue is important because if problems do emerge then the usefulness and importance of the economic theory and econometric methods described comes into question. Secondly, we shall undertake some misspecification analysis. We will compare the simple GTZ notional model to the complex GTZ effective model and see which is more robust to misspecifications. This has important practical implications as regards to modelling in general.

5. An alternative MN specification would be to add errors to (14) and (15) instead of (16) and (17), here separate demand and supply disturbances still exist but lagged endogenous variables will not eventuate. Here a combination of MN disequilibrium and threshold switching methods is needed to formulate likelihood functions.
The effective GTZ specification considered in these simulations has the following form:

\[
\bar{Q}_t^d = Q_t^d + \alpha_2 \left[ \max (0, Q_{t-1}^d - \bar{Q}_{t-1}^d) \right]
\]
(32)

\[
\bar{Q}_t^f = \bar{Q}_t + \beta_2 \left[ \min (0, \bar{Q}_t - Q_t^f) \right]
\]
(33)

\[
\bar{Q}_t^d = \alpha_1 P_t + \alpha_3 X_t^d + \alpha_4 X_{t-1}^d
\]
(34)

\[
\bar{Q}_t^f = \beta_3 P_t + \beta_2 X_t^f + \beta_4 X_{t-1}^f
\]
(35)

\[
Q_t = \min \left( \bar{Q}_t^d, \bar{Q}_t^f \right) + U_t^f
\]
(36)

\[
U_t^f \sim n(0, \sigma_4)
\]
(37)

The corresponding likelihood function is defined in (31) with the definitions \( Q_t^{*w} = \bar{Q}_t^{*w} \) and \( Q_t^{*e} = \bar{Q}_t^{*e} \).

The values assumed for the parameters and the types of exogenous variables correspond to those used in Goldfeld and Quandt (1981). The parameters are: \( \alpha_0 = 18.0, \alpha_1 = -13.0, \alpha_3 = 0.20, \alpha_4 = -0.37, \beta_0 = 0.0, \beta_1 = 3.5, \beta_2 = -0.35, \beta_3 = -0.54, \beta_4 = 0.2 \) and \( \sigma_4^2 = 64.0 \). The regressors follow uniform distributions: \( P_t \sim U(2, 10) \), \( X_{t-1}^d \sim U(20, 90) \), \( X_{t-1}^d \sim U(40, 120) \), \( X_t^f \sim U(1, 7) \), and \( X_{t-1}^f \sim U(135, 400) \). Once generated these are held fixed across all replications, this mimics the fixed regressor assumption. The values for the parameters and regressors describe the generated model’s \( R^3 \)'s. Given the switching between notional and effective functions four \( R^3 \)'s exist: notional demand (0.96), effective demand (0.97), notional supply (0.80) and effective supply (0.89).

The experiments are based on thirty replications. The sample size used is fifty. Since the regressors are held fixed then the number of observations going to the different regimes remains constant, i.e., notional demand (22), effective demand (9), notional supply (14) and effective supply (5).

In optimising the log of (31) step functions must be approximated. In this study we use the Tisher and Zang (1979) spline approximation \( D_4 (r) \), p. 263) this is also used by Sneessens (1985). Following Sneessens (1985, pp. 117-18) a two step procedure is employed, first the approximation is taken large to smooth out the function, then it is taken small to home in on the local optimum. It is argued that the first step is necessary given the multi modal nature of the log of (31).

To investigate computational feasibility two models were generated and
DISEQUILIBRUM ESTIMATION

estimated. First, the model defined by (32)-(37) was generated and the log of (31) optimised to gain estimates. Second, the model defined by (32) — (37) but with $\alpha_3 = \beta_3 = 0$ generates the data and the simple GTZ likelihood function (i.e., (31) but only using $X_{ni}$) is optimised to gain estimates. Here the notional model generates the data and estimates the parameters. The second experiment is necessary to discuss notions of relative computational feasibility.

The results of these two sampling experiments are presented in Table 1. In this table three measures are presented.

Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True : Effective</th>
<th>Estimated : Effective</th>
<th>True : Notional</th>
<th>Estimated : Notional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PMB</td>
<td>PRMSE</td>
<td>PBS</td>
<td>PMB</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>3.73</td>
<td>45.93</td>
<td>56.67</td>
<td>2.79</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.51</td>
<td>-6.92</td>
<td>57.67</td>
<td>0.13</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>3.79</td>
<td>-18.61</td>
<td>43.33</td>
<td>2.99</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.76</td>
<td>4.89</td>
<td>53.33</td>
<td>0.38</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>1.67</td>
<td>11.20</td>
<td>60.00</td>
<td>1.04</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>4.19</td>
<td>60.04</td>
<td>43.33</td>
<td>11.52</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-22.99</td>
<td>-218.42</td>
<td>66.67</td>
<td>-14.98</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-4.91</td>
<td>12.61</td>
<td>50.00b</td>
<td>-3.19</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-19.96</td>
<td>26.17</td>
<td>13.33b</td>
<td>-15.86</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>17.95</td>
<td>196.20</td>
<td>50.00</td>
<td>-</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>15.92</td>
<td>-51.39</td>
<td>36.67</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Since the true $\beta_0 = 0$ then the mean bias and root mean square error are presented.
(b) Denotes significant bias.

6. The optimisation of all log-likelihood functions was undertaken using the Davidon-Fletcher-Powell algorithm. Goldfeld and Quandt's GQOPT4 FORTRAN programs were employed using numerical derivatives. Accuracy in results was set up to 12 decimal places and no computational errors were reported. The approximation coefficient in the spline function was set to 70 in the first instance and 0.0001 in the second. The IMSL package was employed to generate uniform regressors (DRNUN) and the normal error terms (DRNNOA).
\[ PMB : \text{percentage mean bias} \]
\[ = 100 \left\{ \frac{1}{30} \sum_{i=1}^{30} (\hat{\alpha}_i - \alpha)/30 \right\}/\alpha. \]

\[ PRMSE : \text{percentage root mean square error} \]
\[ = 100 \left\{ \frac{1}{30} \sum_{i=1}^{30} (\hat{\alpha}_i - \alpha)^2/30 \right\}^{1/2}/\alpha. \]

\[ PBS : \text{percentage bias statistic} \]
\[ = 100 \{K/30\}. \]

Here, \( \hat{\alpha} \) represents the parameter estimate in the \( i \)th replication, \( \alpha \) represents the true parameter value and \( K \) is the number of times (out of 30 replications) the estimated parameter is greater than the true parameter.

Following Snecssens (1985, p. 121) \( PBS \) can be used to test for insignificant bias. If \( PBS \) is 50 then no bias exists, deviations above 50 imply some positive bias, below 50 some negative bias. Assuming our distributions are symmetric and viewing each of the 30 replications as a Bernouilli trial, then De Moivre's theorem implies that we do not have significant bias (at a 5\% level of significance) if 32.16 \( \leq PBS \leq 67.84 \).

In a general context note that both estimators perform well. The effective model has only 4 out of 10 biases greater than 10\%, 5 out of 10 \( PRMSE \)'s greater than 30\% and only 2 out of 11 significant biases. The relative poor performance of \( \beta_2 \) is somewhat expected given the very few observations which go to these regimes (i.e., 9 and 5 respectively). The notional model has 3 out of 8 biases greater than 10\%, 3 out of 8 \( PRMSE \)'s greater than 30\% and only 1 out of 9 significant biases. In both models the greater bias and variation occurs with supply parameters, this is expected given the fewer supply observations and smaller \( R^2 \)'s. Thus overall in an absolute sense, both models perform well if the true model and estimator coincide.

Next, compare the performances of the effective and notional models. Of the 9 possible mean bias comparisons the notional model is better 8 times. Of the 9 possible \( RMSE \) comparisons the notional model is better 5 times. If we sum (the absolute value of) all comparable \( PRMSE \)'s (we cannot sum \( \beta_2 \), \( \alpha_2 \) and \( \beta_1 \)) then the effective sum of 393.59 is smaller than the notional sum of 400.84; averages of 49.2 and 50.1. Therefore, the notional model seems to perform better in terms of bias, with the effective model performing slightly better in terms of \( RMSE \). The conclusion follows, the effective model is com-
putationally feasible and hence modellers cannot ignore it for computational reasons.

Turn to issues of misspecification. Is it better to misspecify using the effective or notional model if one must misspecify? Two further sampling experiments are undertaken. Table 2 presents for a notional model generating process but an effective model estimator, and for an effective model generating process but a notional model estimator. All other characteristics are as before.

The results are striking. The effective estimator has 2 out of 8 biases greater than 10%, 3 out of 8 RMSE's greater than 30% and 2 out of 11 significant biases. The notional estimator has 3 out of 8 biases greater than 10%, 4 out of 8 PRMSE's greater than 30% and 5 out of 9 significant biases. Of the 9 possible comparisons of biases the effective estimator is better 6 times, of the 9 possible comparison of PRMSE's the effective model is better 4 times. The sum of comparable PRMSE's is much smaller for the effective estimator 418.63

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True : Notional Estimated : Effective</th>
<th>True : Effective Estimated : Notional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PMB</td>
<td>PRMSE</td>
</tr>
<tr>
<td>$a_0$</td>
<td>2.87</td>
<td>48.14</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.34</td>
<td>-7.06</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.67</td>
<td>-20.91</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.51</td>
<td>4.65</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.49</td>
<td>10.13</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>13.79</td>
<td>60.84</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-6.40</td>
<td>-237.64</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-3.97</td>
<td>12.24</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-19.53</td>
<td>27.15</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.049a</td>
<td>0.304a</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>-0.020a</td>
<td>0.048a</td>
</tr>
</tbody>
</table>

(a) Since the true $\beta_0 = 0$ (and $a_3 = \beta_3 = 0$ under true notional) then the mean bias and root mean square error are presented.
(b) Denotes significant bias.
compared to the notional estimator 474.58; averages of 52.33 and 59.32 respectively.

The clear conclusion is to misspecify with the effective model rather than the notional model. The consequences of misspecification for the notional model can be dire. 56% of parameters have statistically significant bias. These results are somewhat expected as the effective model is the general model in which the notional model is nested. Note, the parameters $\alpha_2$ and $\beta_3$ performed quite well in Table 2 minicing $\alpha_2 = \beta_3 = 0$ very closely. Thus as per conventional linear regression theory, the consequences of including an irrelevant variable are not as serious as the consequences of excluding a relevant variable.

6. CONCLUDING COMMENTS

This paper has reconciled single market effective demand theory with single market disequilibrium econometrics. Some new ideas on theory have been presented, the new representation has focused on untapped aspects of actual rationing. It is claimed that the concept of agent asymmetry is needed to reconcile deterministic manipulability with actual rationing. The representation has derived effective demand from optimising principles and has related the difference between trade and demand to agent dissatisfaction.

In attempting to apply this theory, it seems that only the GTZ specification with non-perfect expectations is suitable. Perfect expectations suffer from both conceptual and non-estimability problems. The MN specification with non-perfect expectations is computationally intractable.

The GTZ specification with one period lagged expectations was shown to be computationally feasible. In single applied studies its performance will be much improved given the liberty to employ many different algorithms and approaches; in this study given the Monte Carlo approach and costs, we were limited to using only one algorithm with numerical derivatives. The consequences of misspecifying the effective model when the generation process is the notional model are minor. On the other hand, the consequences of misspecifying the notional model when the effective model generates the data were shown to be serious: more than half the parameters are estimated with significant bias. In a general modelling context, if unsure as to which specification to employ, then use the effective model, it is computationally feasible and the consequences of misspecification are relatively mild.
REFERENCES


