Referee assignment in sports leagues: approximate and exact multi-objective approaches

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1 Introduction

Amateur leagues of several sports frequently face the problem of assigning referees to the previously scheduled games of a competition. This problem appears e.g. in some regional sport leagues, such as baseball, basketball, and soccer. Hundreds of games are played every weekend in different divisions. For instance, in the MOSA (Monmouth & Ocean Counties Soccer Association) league, New Jersey, children of ages 8 to 18 make up six divisions per age and gender group with six teams per division, totaling 396 games every Sunday, which are officiated by hundreds of certified referees.

Sport games are regulated by rules that depend on the sport and tournament. The officiating crew is a group of referees that is responsible to ensure that all rules are respected in a game. The number of referees compounding a crew may vary, depending on the sport, league, and tournament. Each member of an officiating crew fills a refereeing position in a game. The Referee Assignment Problem (RAP)\textsuperscript{1,2} consists in assigning referees to the refereeing positions for all games of a league or tournament.

Referee assignment is subject to a number of constraints and optimization criteria. Games in higher divisions may require higher-skilled referees. Due to the shortage of certified referees, each of them may officiate several games in the same day. In some applications, each referee declares the target number of games he/she is willing to officiate and a possible objective consists in minimizing the sum of the absolute value of the difference between the target and the actual number of games assigned to each referee. Tournament organizers may also want referee assignments matching preferences regarding the facilities, divisions, and time slots where the referees officiate. The minimization of the idle times between consecutive games assigned to the same referee is another possible objective. The referee assignment problem has clearly the flavor of a multi-criteria optimization application. Referee assignment in other contexts has been addressed e.g. in [5,6,8].

This work investigates the solution of a bi-objective version of the referee assignment problem, which is stated in the next section. Section 3 describes the proposed solution strategy and some preliminary results. Concluding remarks are drawn in the last section.

2 Problem statement

We consider the general problem, in which each game has a number of refereeing positions to be assigned to referees. The games are previously scheduled and the facilities and time slots where they take place are known beforehand. Referees are assigned to empty refereeing positions. This approach allows not only to handle referee assignment problems in different sports, but also problems in tournaments where different games may need different numbers of referees. Each refereeing position to be filled by a referee is called a referee slot.

Each referee slot has to be filled by a referee with a minimum skill level, which is previously determined and often related to the tournament division. Usually, a division corresponds to a set of teams formed by players under a certain age and with the same gender, e.g. boys under 16 years old. Each referee has a certain skill level defining the games he/she can officiate. Additionally, referees may declare their unavailability to officiate at certain time slots. Furthermore, each referee establishes the maximum number of games he/she is able to officiate and the target number of games he/she is willing to officiate. Travels are not allowed, i.e. referees that officiate more than one game in the same day must be assigned to games that take place at the same facility.

The bi-objective referee assignment problem (biRAP) consists in assigning referees to all referee slots associated to games scheduled to a given time interval (typically, a day or a weekend), minimizing (i) the sum over all referees of the absolute value of the difference between the target and the actual number of games assigned to each referee and (ii) the sum over all referees of the idle times between consecutive games assigned to the same referee, and subject to the constraints: (1) all referee slots must be filled for
all games; (2) referees cannot officiate games in
deferree slots overlapping time slots where they are
already scheduled to officiate; (3) referees cannot
officiate games in deferree slots where they de-
clared to be unavailable; (4) referees must meet
the minimum skill level established for each ref-
eree slot; (5) referees cannot officiate more than
a given maximum number of games; and (6) re-
fees cannot officiate in two or more different fa-
cilities on the same day.

3 Solution strategy

Problem biRAP can be formulated as a bi-
objective set partitioning problem, in which each
decision variable corresponds to a sequence of re-
deere slots assigned to the same referee. The com-
plete minimum Pareto set can be computed for
medium-size problems by dichotomic search [4]
using a commercial solver such as CPLEX 9.0.
Results for an instance with 50 games and 100
 referees are displayed in Figure 1.

![Graph](image)

**Fig. 1.** Minimum set of efficient solutions for an
instance with 50 games and 100 referees.

To find good approximations of the Pareto set
for real-size instances, we propose a heuristic ap-
proach based on extensions of the three-phase
strategy (a greedy constructive procedure, a re-
pair heuristic to make solutions feasible, and a hy-
brid iterated local search [7] improvement heuristic
with an embedded mixed integer program-
ing strategy) presented in [1], originally de-
veloped for the single objective referee assignment
problem. This approach consists in iteratively ex-
ecuting an adaptation of the three-phase heuristic
for a given set of optimization directions. Each
phase of the three-phase heuristic was adapted
to search for potentially efficient solutions for bi-
RAP, considering a scalarized objective function
and given the current search direction [3]. The
algorithm solves a MIP problem at each iter-
ation of the adapted ILS heuristic, considering
the scalarized objective function and following
the current search direction. The approximation
method merges the sets of locally potentially ef-

cient solutions found by each execution to form
the final set of potentially efficient solutions [4].

The results obtained by the approximation
method are compared with the complete mini-
um set of efficient solutions found by the ex-
act procedure, illustrating the effectiveness of the
proposed approach. Additional experiments dis-
cuss further extensions, such as a path relinking
procedure.

4 Concluding remarks

We formulated the new bi-objective referee
assignment problem. The complete minimum
Pareto set was computed for medium-size prob-
lems by the dichotomic method. A promising
heuristic approach is based on the ILS metaheur-
istic and using an embedded MIP strategy was
presented. Preliminary results illustrate the effec-
tiveness of the approach in the solution of real-
size instances. We are currently working on ex-
tensions addressing further constraints and objec-
tives of real-life applications, such as the min-
imization of inter-facility traveling times.

References

1. A.R. Duarte, C.C. Ribeiro, and S. Urrutia. A hy-
brid ILS heuristic to the referee assignment prob-
lem with an embedded MIP strategy. Lecture
Haeusler. Referee assignment in sport leagues.
Lecture Notes in Computer Science, to appear.
3. M. Ehrgott. Multicriteria Optimization. Springer-
Verlag, 1999.
4. M. Ehrgott and X. Gandibleux. Approximative
solution methods for multiobjective combinato-
5. J.R. Evans. A microcomputer-based decision sup-
port system for scheduling umpires in the Ameri-
umpire crews for professional tennis tournaments.
local search. In F. Glover and G. Kochenberger,
editors, Handbook of Metaheuristics, pages 321–
Journal of the Operational Research Society,