A THEORY OF MARKET QUANTITY CONTROLS: THE USE OF DISEQUILIBRIUM AND BARGAINING THEORIES

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I. INTRODUCTION

This paper attempts to outline a new theoretical approach for analyzing the imposition of a quantity control on a single exchange trading market. To facilitate this analysis of market quotas, we shall employ the single market 'disequilibrium' notions of Benassy (1982) and others, and the economic bargaining theory of Zeuthen (1930), Nash (1950, 1953), and Harsanyi (1956) (hereafter labelled the ZNH bargaining theory). In part, the ideas 'hinted at' by authors such as John Stuart Mill in 1869 (Mill, 1967, p. 642) and Chambers et al. (1981, p. 133) are formalized and expanded. That is, it is recognized that multiple price equilibria exist at the quota level and hence unique price resolution may depend upon bargaining.

Section II outlines and dismisses the traditional textbook analysis of market quotas and so proposes an alternative framework based on effective demand/supply behavioural postulates. Multiple price equilibria is predicted from these new postulates and hence alone effective demand/supply theory provides no unique price theory for markets with quotas. In response, Section III investigates the bargaining theoretic resolution to unique price by interpreting the multiple equilibria range as a bargaining range. The ZNH bargaining model is therefore employed with conditional indirect utility functions (which measure preferences in terms of price for a given fixed quantity) to predict a unique price solution.

Section IV presents a specific illustrative example of how one might relate the bargaining solution formulated in terms of conditional indirect utilities, back to direct utility functions and hence notional demand/supply functions. This will provide a complete and logically consistent picture of this effective demand/supply -- bargaining approach to the analysis of market quotas. Section V closes with some concluding comments.

II. QUANTITY CONTROLS AND DISEQUILIBRIUM THEORY

Consider the conventional textbook analysis of a binding quantity control (for example Hirschleifer, 1976, pp. 188-9), diagrams (such as Figure 1) are used to facilitate the analytical discussion.

*Thanks go to A. S. G. Lubulwa who read an earlier draft of this paper, and to M. P. Schneider who pointed out the relevance of John Stuart Mill's work to this discussion.
In Figure 1 standard notional demand/supply curves ($Q^d$ and $Q^s$) are drawn within the quantity-price space. Notional equilibrium price and quantity are depicted as $P^e$ and $Q^e$ respectively. Assume that the binding quota imposed restricts quantity from rising above $\bar{Q}$, this restriction provides two intersections with demand/supply curves, $(\bar{Q}, \bar{P}^d)$ and $(\bar{Q}, \bar{P}^s)$, two conflicting prices arise. Generally, it is assumed that the transacting price will be "at whatever price the market will bear," hence due to the excess of demand price over supply price, the price transacted is $\bar{P}^d$.

Two interrelated objections can be laid against the above analysis. First, the theory underlying notional demand/supply curves is no longer relevant given the quantity control. Clearly, if $\bar{Q} < Q^e$ then $\bar{Q}^d \neq Q^s$, disequilibrium in notional desires exists by definition. Thus, both buyers and sellers simultaneously cannot buy and sell as much as they desire at any given price, and as such the conditions for notional optimisation are violated. In response new optimisation programs need formulation to account for the new quantity controlled situation; this is considered later in this section.

Secondly, given the predicted transacted co-ordinate $(\bar{Q}, \bar{P}^d)$ and the knowledge of this market process to all agents, then it seems intuitively naive to predict that demanders will continue to pay $\bar{P}^d$ for $\bar{Q}$. It is immediately obvious that even at a price of $\bar{P}^s$ sellers are willing to trade $\bar{Q}$. Clear savings can be made by buyers, therefore, if they collectively act to
bargain for a lower trading price.\textsuperscript{1} Such bargaining resolutions of price are treated in Section III.

How might we go about defining new behavioural postulates for quota infested markets? Fortunately, with the recent emergence of literature on the ‘markets in disequilibrium’, a suitable framework lies in wait. Authors such as Grossman (1974), McCafferty (1977) and Benassy (1982, chs 1-3) have provided analyses of single market situations characterised by disequilibrium trading, thereby formalising the notion of effective demand/supply. In that literature, rationing aspects of disequilibrium trading are highlighted and thus notions, such as the manipulability of rationing schemes, perceptions of rationing and transaction costs associated with expressing demands are discussed.

In this analysis of quotas the following rationing schema is assumed. There exists a regulating authority who sets and administers the quotas. This authority acts as the rationing distributor and determines exactly (and sets legally) the maximum amount of the commodity each demander/supplier can transact. It would seem that non-manipulable rationing schemes with perfectly perceived rationing expectations best describe such quota markets. Clearly, if quotas are legally enforceable then manipulation through demand/supply is impossible, that is, no matter how high demand/supply is set, at best agents receive only the quota amount in return. Further, since this quota amount is announced in advance to each agent before trading, then each agent knows perfectly and with certainty how their demands/supplies will turn into transactions.\textsuperscript{2}

Unfortunately, a formal representation of the just described market process will leave effective demand/supply formulation cloudy and inoperative. As Benassy (1982, p. 33) suggests, since any expressed demand/supply at or above the quota level will produce a quota transaction then an infinite number of demands/supplies will be utility maximising and hence, could be labelled effective demand/supply; a ‘problem in definition’. Such a result is certainly undesirable, but a simple resolution can be offered.

\textsuperscript{1}Mill (1967, p. 642) discusses the notion of collective action by buyers and sellers in the following terms:-

“When, therefore, several prices are consistent with carrying off the whole supply, the dealers are tolerably certain to hold out for the highest of these prices; for they have no motive to compete with one another in cheapness, there being room for them all at the higher price. On the other hand, the buyers are not compelled by each other’s competition to pay that higher price; for if the buyers hold out for a lower price and get it, their gain may be permanent”.

In many agricultural markets, large numbers of agents do exist at the farm level, yet at trading places collective behaviour is represented by producer collectives, associations and marketing boards. Thus it seems plausible to assume the existence of many small agents who have no individual influence over price, and collective organisations who can manipulate price on behalf of agents.

\textsuperscript{2}One can find many ‘real world’ examples of markets which have non-manipulable, (pre-trading) announced quotas. The Australian tobacco growing industry (see Australian Tobacco Board, Annual Reports) and the U.S. import beef market (see Chambers et al., 1981) are two examples. Note, the introduction of stochastic and/or manipulable quotas introduces much more complexity and necessarily requires a totally different approach to that given here i.e. bargaining notions no longer seem relevant. This comment does not diminish the usefulness of our non-manipulable announced quota results as markets with these characteristics do in fact exist.
Under the above circumstances, it remains cost free to express demands/supplies. However, as Grossman (1974), McCafferty (1977), Svensson (1981) and others point out, one can list many costs as testament to the existence of demand/supply dependent transaction costs. Costs are associated with all the following activities: searching and shopping; applying for jobs; market representatives; waiting in a queue; delivery delays; demand/supply expression taxes; market operations in the collation and recording of bids, etc. Given such demand/supply expression transaction costs, it immediately follows that effective demand/supply equal to the quota level will be optimal; all higher bids simply incur a cost and no extra transaction. As Svensson (1981, p. 3) puts it, "... clearly under these circumstances there is no point in agents expressing any demands in excess of the constraints".

A formal justification of these claims now follows. Consider a single market model of pure exchange, assume the existence of many buyers and sellers, a perfect competition market structure. Demanders and suppliers are assumed to have preferences described by the following utility functions:

\begin{align*}
\text{Demanders:} & \quad U(x^d, m^d) \\
\text{Suppliers:} & \quad Z(x^s, m^s) \\
& \quad \text{with} \\
& \quad x^d = w^d + Q \\
& \quad m^d = \bar{m}^d - PQ \\
& \quad x^s = w^s - Q \\
& \quad m^s = \bar{m}^s + PQ \\
& \quad \text{and} \\
& \quad x^d, x^s \quad \text{final holdings of the good} \\
& \quad m^d, m^s \quad \text{final holdings of money} \\
& \quad w^d, w^s \quad \text{endowments of the good} \\
& \quad \bar{m}^d, \bar{m}^s \quad \text{endowments of money} \\
& \quad P, Q \quad \text{transaction price and quantity of the good}
\end{align*}

This representation supersedes the existence of other commodities along the lines of the composite commodity formulation. Both \(x\) and \(m\) derive utility, \(m\) through its store of value and the implied supressed purchases of other goods. It is clear that for given endowments and prices the substitution of (2) into (1) leaves utilities as a function of \(Q\) only. It is assumed that \(U()\) and \(Z()\) are strictly concave in \(Q\) and that the unconstrained maximisation of (1) defines unique notional demands and supplies of \(Q^d\) and \(Q^s\) respectively.

To determine effective demand \(Q^d\) and effective supply \(Q^s\), assume the existence of a

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3Given the peculiar non-manipulable rationing scheme where the rationing distribution is determined independently of specific individual agents then the need to distinguish between individual and aggregate agent behaviour is unnecessary. Therefore, the formal analysis about to be undertaken is representative of both individual and aggregate agent behaviour.
quota $\bar{Q}$. Given the assumptions of non-manipulability and perfect rationing expectations, then the relation between demands/supplies and transactions can be written as:

$$Q = \min (Q^d, \bar{Q})$$

$$Q = \min (Q^s, \bar{Q})$$

(3)

Demand and supply dependent transaction costs can be measured as:

$$K(Q^d) \text{ if } Q^d > 0$$

$$G(Q^s) \text{ if } Q^s > 0$$

(4)

where $K(.)$ and $G(.)$ are strictly increasing in $Q^d$ and $Q^s$ respectively, that is, it costs a positive utility sum to place a bid for each single unit.  

It is clear that effective demands/supplies are defined by choosing $Q^d$ and $Q^s$ which maximise (1) minus (4) subject to (2) and (3). That is, transaction costs enter negatively into the objective function and (3) describes the new quantity control which constrains optimisation. Results directly follow. For demand there exist two solution ranges. If $\bar{Q} \geq Q^d$ then the quota is not binding and notional demand will be expressed; $Q^d = \bar{Q}$ and $Q^d < \bar{Q}$ then the quota is binding and $Q^d = \bar{Q}$, clearly this avoids the utility wastage for any $Q^d > \bar{Q}$ as measured by $K(Q^d) - K(\bar{Q})$. Similar arguments define effective supply as: if $\bar{Q} \geq Q^s$ then $Q^s = \bar{Q}$, if $\bar{Q} < Q^s$ then $Q^s = \bar{Q}$.

In the endeavour to determine transacting price, effective demand/supply schedules (i.e. desires for all possible price levels) need deriving. Refer back to Figure 1. Clearly, effective demand for high price follows $\bar{Q}^d$ until $\bar{p}^d$, at which point it drops vertically at $\bar{Q}$ until $p = 0$. Effective supply for low price follows $\bar{Q}^s$ until $\bar{p}^s$, at which point it rises vertically at $\bar{Q}$ until $p = \infty$.

If left to market forces, then price is over-determined, that is, there exist a whole range of prices ($\bar{p}^d \geq p \geq \bar{p}^s$) at which effective demand equals effective supply. Therefore, if we carry through our small agent perfect competition market structure assumption and assume that an independent auctioneer sets and adjusts price, then the choice between $\bar{p}^d$ and $\bar{p}^s$ must necessarily be arbitrary. It is for these reasons that bargaining resolutions of unique price are investigated in the next section.

### III. Quantity Controls and Bargaining

If the quota amount is to be traded, then the multiple equilibria range might be best described as a bargaining range, i.e., demanders preferring a low price, suppliers a high price. Preferences in terms of price therefore, need formulation, hence use is made of conditional indirect utilities (see Cornes and Albon, 1981, p. 188). Since $\bar{Q}$ is to be traded, then $Q$ must be kept fixed throughout in defining price preferences, thus the substitution of

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*Rather than impose costs as deductions from money balances $m$, for ease of exposition and clarity of descriptions we explicitly measure costs in utility terms directly. This approach is taken by Benassy (1982, pp. 179-80).*
\( Q = Q \) into (2) and eventually into (1) defines conditional (on fixed quantity) indirect utility functions as:

\[
V(\omega^d + Q, \bar{m}^d - P\bar{Q}) \\
W(\omega^s - \bar{Q}, \bar{m}^s + P\bar{Q})
\]  

(5)

\( V(\cdot) \) is strictly decreasing in price, while \( W(\cdot) \) is strictly increasing in price. Consider demand, given the fixed indirect utility from \( \bar{Q} \), lower prices increase money balances and through its store of value, higher money balances increase utility.

The bargaining theory employed is the well known and used Zeuthen-Nash-Harsanyi theory. Emphasis is made of the Zeuthen concession making interpretation of bargaining. Define the two market players by DR, a demanders' representative and SR, a suppliers' representative. Assume further that the quota regulator acts also as an arbitrator who oversees negotiations.

Zeuthen\(^1\) suggests that bargaining follows a series of offers and counter offers (i.e. \( P^{d}_{1,2}, \ldots P^{d}_{i,j}, \ldots \) for DR and \( P^{s}_{1,2}, \ldots P^{s}_{i,j}, \ldots \) for SR), with respective sides conceding on bids until eventually either a deadlock or agreement is reached. The specific concession making process is said to depend on the comparison of maximum probabilities of conflict, i.e. \( C^{d}_{1,2}, \ldots C^{d}_{i,j}, \ldots \) for DR and \( C^{s}_{1,2}, \ldots C^{s}_{i,j}, \ldots \) for SR. At each stage, \( C^{d}_{i,j} \) and \( C^{s}_{i,j} \) are compared and concessions proceed according to which side is less willing to risk conflict for example, if \( C^{d}_{i} < C^{s}_{i} \), then DR is less willing to risk conflict and hence raises its offer of \( P^{d} \). This conceding continues until eventually \( C^{d}_{i,j} = C^{s}_{i,j} = 0 \) at which point agreement is made, \( P^{s} = P^{d} \). Or equivalently, as Nash has shown, agreement will be reached when the product of utility increments (as measured from the expected conflict payoffs) is maximised:

\[
\text{Max } [V(P, \cdot) - V(CF)] [W(P, \cdot) - W(CF)]
\]  

(6)

where \( V(CF) \) and \( W(CF) \) represent the expected utility conflict payoffs.\(^6\)

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\(^1\)For an excellent recent description and summary of the following results see Sapsford (1982, pp. 8-15).

\(^6\)To ease the subsequent exposition, the transaction costs defined in (4) are assumed to be: \( K(Q^{d}) = 0 \) if \( Q^{d} < \bar{Q} \), otherwise \( K(Q^{d}) > 0 \); and \( G(Q^{s}) = 0 \) if \( Q^{s} < \bar{Q} \), otherwise \( G(Q^{s}) > 0 \). That is, if individual agents do not attempt to gain more than their legally allocated amount then no costs are incurred. On the other hand, the only possible hope of gaining more than legal rations is through searching and lobbying, which is necessarily costly. It is the assumption that all manipulative attempts are expected to be unsuccessful but costly, which produces \( Q^{d} = Q^{s} = \bar{Q} \). This plausible assumption is sufficient to establish the multiple equilibria discussed in Section II. The assumption also emphasizes that the role of transaction costs is just to establish that bargaining is necessary and that they play no significant role in the bargaining process itself. Thus, in the remainder of the paper transaction costs are ignored and not subtracted from (5). The incorporation of non-zero costs for \( Q^{d} = Q^{s} = \bar{Q} \) unnecessarily complicates the expected conflict utilities and as argued above, adds nothing of real substance. In fact, the problem (6) and solution (7) are unaffected by non-zero costs, that is, only the expected conflict utility is affected, the utility increments (measured from the conflict position) remain unaffected. In other words, all the subsequent results based purely on incremental utilities are still valid with non-zero costs for \( Q^{d} = Q^{s} = \bar{Q} \).
The first order optimality condition stemming from (6) requires the bargaining price solution \( P^*_b \) to satisfy:

\[
\frac{V(P^*_b \cdot \cdot)}{W(P^*_b \cdot \cdot)} = \frac{V(P^*_b \cdot \cdot) - V(CF)}{W(P^*_b \cdot \cdot) - W(CF)}
\]

(7)

In words, \( P^*_b \) is chosen such that the elasticity of the utility increments frontier equals minus one. Note, if the participants are identical in very respect then the solution predicts:

\[
V_j(P^*_b \cdot \cdot) = W_j(P^*_b \cdot \cdot)
\]

(8)

that is, equal utility increments. This also implies equal total utility payoffs given that \( V(CF) = W(CF) \).

To develop further results expected conflict payoffs need specifying. Given the empirical reality that quota amounts are always traded then the no agreement payoff must relate to a trade situation. Assume that if conflict, then the regulator acts as an arbitrator and sets a price \( P^a \) at which \( Q \) will be traded; assume that \( \overline{P}^d \geq P^a \geq \overline{P}^s \), this ensures that both DR and SR are still willing to trade \( Q \).

Consider first the perfect expectation assumption, that both DR and SR realise \( P^a \) to be the conflict price. The conflict payoff utilities become: \( V(CF) = V(P^a \cdot \cdot) \) and \( W(CF) = W(P^a \cdot \cdot) \). Under such circumstances, it is clear that \( P^a \) will be the trading price. Consider the following argument. Assume the initial bids of \( P^d_1 \) and \( P^s_1 \), clearly, \( V(P^d_1 \cdot \cdot) > V(P^a \cdot \cdot) \) and \( W(P^d_1 \cdot \cdot) > W(P^a \cdot \cdot) \), and \( P^d_1 < P^a < P^s_1 \) this indicates that positions better than the conflict payoffs are desired. Concessions take place until either \( P^d_1 = P^a \) or \( P^s_1 = P^a \). Consider the former \( P^d_1 = P^a \) here the maximum probability of conflict for DR (i.e. \( C^d_1 \)) approaches infinity, that is, DR will concede no further. In other words, at \( P^d_1 = P^a \) the expected conflict payoff is reached, further concessions will only produce payoffs less than those expected in conflict, hence conceding ceases. In such a situation \( C^d_1 > C^s_1 \) arises implying that SR will unilaterally concede, lowering \( P^s_1 \) until \( P^s_1 = P^d_1 = P^a \) at which point \( C^d_1 = C^s_1 = 0 \). Similar arguments hold if \( P^s_1 = P^a \) occurs before \( P^d_1 = P^a \).

It is somewhat unrealistic and theoretically meaningless to assume that this conflict/arbitration price is known before negotiations get under way because, as we have shown, in such circumstances negotiations serve no purpose. Alternatively, it seems preferable to assume that information about \( P^a \) is scarce and that expectations need formation, i.e. DR expects \( \overline{P}^d \) and SR expects \( \overline{P}^s \).

First, it is clear that such expectations must satisfy: \( \overline{P}^d \geq P^d_1 \geq P^a \) and \( \overline{P}^s \leq P^s_1 \leq P^a \), that is, all offers and counter offers will relate only to improvements upon (or equality with) the expected conflict payoff and that these expectations not contradict their utility maximisation effective demand/supply trade positions. This latter requirement stems from both sides’ knowledge that the arbitrator will not set \( P^a \) at any level which will violate utility maximisation for the trade of \( Q \).
Assume the existence of "pessimistic" players. DR realises that the arbitrator will not set $p^d$ above $\bar{p}^d$ (otherwise DR no longer desires to trade $\bar{Q}$), pessimistically therefore $p^{ad} = \bar{p}^d$. Note also that this expectation is the only one which attaches no risk, any other expectation ($p^{ad} < \bar{p}^d$) permits the risk of a worse actual conflict price. Similarly, SR's pessimistic expectation is $p^{as} = \bar{p}^s$. In the previous terminology; $V(CF) = V(\bar{p}^d, \cdot)$ and $W(CF) = W(\bar{p}^s, \cdot)$.

It is clear that given $\bar{p}^d > \bar{p}^s$, then such schema permit the conventional ZNH solution as represented by (7). Here the product of utility increments as measured from $V(\bar{p}^d, \cdot)$ and $W(\bar{p}^s, \cdot)$ will be maximised. Obviously, $\bar{p}^d \geq p^*_b \geq \bar{p}^s$, DR will not concede above $\bar{p}^d$ and SR not below $\bar{p}^s$. Further results can be gained in the special case of strict symmetry, i.e. $V(\bar{p}^d - A, \cdot) = W(\bar{p}^s + A, \cdot)$ and $-V(\bar{p}^d - A, \cdot) = W(\bar{p}^s + A, \cdot)$ for all $A \geq 0$. In words, utilities at the conflict prices and at any equi-distance price departures from conflict prices are assumed to be equal, as are the absolute values of the associated first derivatives. Under the assumption of $V_i(\bar{p}^d - A, \cdot) = W_i(\bar{p}^s + A, \cdot)$ equation (8) imposes the requirement that $\bar{p}^d - A = \bar{p}^s + A$, i.e. $A^* = (\bar{p}^d - \bar{p}^s)/2$. Thus under strict symmetry the bargaining price is $p^*_b = \bar{p}^d - A^* = \bar{p}^s + A^* = (\bar{p}^d + \bar{p}^s)/2$.

Asymmetric results can be obtained by making specific functional form assumptions about $V(\cdot)$ and $W(\cdot)$. Consider the linear forms:

$$V(P, \cdot) = a + b \:(P - \bar{p}^d)$$

$$W(P, \cdot) = c + d \:(P - \bar{p}^s)$$

where $a \geq 0, b < 0, c \geq 0, d > 0$, $a$ and $c$ represent the expected conflict utility payoffs. The application of equation (7) here implies:

$$p^*_b = \frac{\bar{p}^d + \bar{p}^s}{2}$$

Note, this is exactly the result obtained in the symmetric case. However, in terms of utility increments (measured from $a$ and $c$) the bargaining payoffs are:

$$V_i(p^*_b, \cdot) = -b \frac{(\bar{p}^d - \bar{p}^s)}{2}$$

$$W_i(p^*_b, \cdot) = d \frac{(\bar{p}^d - \bar{p}^s)}{2}$$

Clearly, such utilities are only equal if $-b = d$. On the other hand, for $d > |b|$ $W_i(p^*_b, \cdot) < V_i(p^*_b, \cdot)$ and vice versa. In words, the side which is more capable of enjoying equi-distance price changes (from their expected conflict/arbitration prices) receives the higher incremental utility payoff.

Consider an alternative justification of $V(CF) = V(\bar{p}^d, \cdot)$ and $W(CF) = W(\bar{p}^s, \cdot)$ With perfect knowledge of the bargaining range, the arbitrator might impose an ultimatum. If agreement is not reached then demanders pay $\bar{p}^d$, and suppliers receive $\bar{p}^s$, the difference is charged as a tax. Here, if conflict arises then both demanders and suppliers are willing to trade but the possible gains from bargaining accrue to the government.
IV. An Illustrative Theoretical Example

In this section we seek to reconcile such bargaining/arbitration outcomes, determinable with the aid of conditional indirect utility functions, with agents' direct utility preferences and hence notional demand/supply curves. We make a specific direct utility functional form assumption and follow its consequences through to the bargaining resolution of price. In doing this consistent preferences are maintained and a complete picture of market operations is logically constructed.

Make the assumption that agents possess multiplicative direct utility functions:

Demander: \( U(\cdot) = x^d, m^d \) \hspace{1cm} (12)
Supplier: \( Z(\cdot) = x^s, m^s \)

With explicit money and commodity endowments:

\[
U(\cdot) = (w^d + Q)(m^d - PQ)
\]
\[
Z(\cdot) = (w^s - Q)(m^s + PQ)
\]

The unconstrained maximisation of (13) produces the notional demand and supply functions:

\[
\bar{Q}^d = (w^d/2) + (m^d/2)(1/P)
\]
\[
\bar{Q}^s = (w^s/2) - (m^s/2)(1/P)
\]

From (14) notional equilibrium price and quantity are:

\[
p^* = (m^d + m^s) / (w^d + w^s)
\]
\[
Q^* = (w^s m^d - w^d m^s) / [2(m^d + m^s)]
\]

We can now begin the describe the bargaining situation. Assume a quota \( \bar{Q} \) such that \( 0 < \bar{Q} < Q^* \); the inverse of (14) therefore determines the bargaining range limits \( \bar{p}^d \) and \( \bar{p}^s \), i.e.

\[
\bar{p}^d = m^d / (2\bar{Q} + w^d)
\]
\[
\bar{p}^s = m^s / (w^s - 2\bar{Q})
\]

Fixing \( Q = \bar{Q} \), transforms the direct utility functions (13) into conditional indirect utility functions:

\[
V(P, \cdot) = m^d(w^d + \bar{Q}) - \bar{Q}(w^d + \bar{Q})P
\]
\[
W(P, \cdot) = m^s(w^s - \bar{Q}) + \bar{Q}(w^s - \bar{Q})P
\]

The bargaining situations described assume conflict payoffs of \( V(\bar{p}^d, \bar{Q}, \cdot) \) and \( W(\bar{p}^s, \bar{Q}, \cdot) \) respectively. Given these expected conflict payoffs, the conditional indirect
utility functions (17), have linear form as described in Section III by equation (9), with the definitions:

\[ a = (w^d + \bar{Q})(\bar{m}^d - \bar{Q} \bar{p}^d) \]
\[ b = -(w^d + \bar{Q}) \bar{Q} \]
\[ c = (w^s - \bar{Q})(\bar{m}^s + \bar{Q} \bar{p}^s) \]
\[ d = (w^s - \bar{Q}) \bar{Q} \]

(18)

Under such circumstances, the bargaining price is defined by equation (10). At the given quota \( \bar{Q} \), bargaining payoff increments are measured from \( \bar{p}^d \) and \( \bar{p}^s \), or in utility terms, from:

\[ V(\bar{p}^d, \bar{Q}, \cdot) = \bar{m}^d(w^d + \bar{Q}) - \bar{Q}(w^d + \bar{Q})\bar{p}^d \]
\[ W(\bar{p}^s, \bar{Q}, \cdot) = \bar{m}^s(w^s - \bar{Q}) + \bar{Q}(w^s \bar{Q})\bar{p}^s \]

(19)

The incremental utility payoffs of the ZNH solution are:

\[ V_{j}(P_{b}^o, \bar{Q}, \cdot) = (w^d + \bar{Q})\bar{Q}(P_{b}^o - \bar{p}^d) \]
\[ W_{j}(P_{b}^o, \bar{Q}, \cdot) = (w^s - \bar{Q})\bar{Q}(P_{b}^o - \bar{p}^s) \]

(20)

The addition of (19) and (20) defines the total utility payoffs of:

\[ V(P_{b}^o, \bar{Q}, \cdot) = (w^d + \bar{Q})(\bar{m}^d - \bar{Q} P_{b}^o) \]
\[ W(P_{b}^o, \bar{Q}, \cdot) = (w^s - \bar{Q})(\bar{m}^s + \bar{Q} P_{b}^o) \]

(21)

To gain specific results assume that the utility value of non-trade endowments are equal, \textit{i.e.} when \( \bar{Q} = 0 \), \( V(\cdot) = W(\cdot) \), or in other words:

\[ \bar{m}^s w^s = \bar{m}^d w^d \]

(22)

Conveniently, the substitution of (22) into (15) defines equilibrium quantity to be:

\[ Q^o = (w^s - w^d)/2 \]

(23)

at which quantity, total utilities are equal \( i.e. \ V(P^o, \cdot) = W(P^o, \cdot) \).

It is clear that the total utility payoffs (as defined by (21)) depend upon both, the bargaining starting points (as defined by (19)) and the ZNH incremental utility payoffs (as defined by (20)). The following results are easily proven for these quantities:

\[ V(\bar{p}^d, \bar{Q}, \cdot) > W(\bar{p}^s, \bar{Q}, \cdot) \quad 0 < \bar{Q} < Q^o \]

(24)

and

\[ V_{j}(P_{b}^o, \cdot) < W_{j}(P_{b}^o, \cdot) \quad 0 < \bar{Q} < Q^o \]

(25)
That is, demanders always begin bargaining from the better starting point, but suppliers always gain more from the ZNH bargaining process in terms of increments. In the previous linear terminology, $a > c$ and $|b| < d$ always.

Surprisingly (24) and (25) offset each other exactly when $\bar{Q} = Q^e/2$, i.e. $V(P_b^s \cdot \cdot) = W(P_b^p \cdot \cdot)$ if $\bar{Q} = Q^e/2$. This also implies that $P^e = P^s_b$ when $\bar{Q} = Q^e/2$. That is, the notional equilibrium and bargaining prices are equal if the quota is half the equilibrium quantity, at which point total payoff utilities coincide. Apart from these equality results, total utility payoff inequalities can also be described:

$$V(P_b^s \cdot \cdot) > W(P_b^p \cdot \cdot) \text{ if } \bar{Q} > (Q^e/2) \text{ and } P_b^p < P^e$$

$$V(P_b^s \cdot \cdot) < W(P_b^p \cdot \cdot) \text{ if } \bar{Q} < (Q^e/2) \text{ and } P_b^p > P^e$$

(26)

That is, in terms of total utility payoffs, demanders “win” for “high” quantities hence the “low” bargaining price, suppliers “win” for “low” quantities hence the “high” bargaining price.

The above results can be summarised by the often measured quantities of notional price and notional income elasticity. Since we employ a one commodity utility function, then the homogeneity restriction implies that notional income and price elasticities are identical (but for sign). When price elasticities $|\varepsilon_d|$ and $\varepsilon_s$, and income elasticities $\eta_d$ and $\eta_s$, for notional demand and supply respectively, are evaluated for a given $Q$ (i.e. at $Q = \bar{Q}$ the magnitudes are calculated for $P = \bar{P}^d$ and $P = \bar{P}^s$ respectively), then the results discussed above can be summarised:

$$V(P_b^s  \cdot \cdot) > W(P_b^p  \cdot \cdot) \text{ if } |\varepsilon_d| > \varepsilon_s \text{ or } \eta_d > \eta_s$$

$$V(P_b^s  \cdot \cdot) = W(P_b^p  \cdot \cdot) \text{ if } |\varepsilon_d| = \varepsilon_s \text{ or } \eta_d = \eta_s$$

$$V(P_b^s  \cdot \cdot) < W(P_b^p  \cdot \cdot) \text{ if } |\varepsilon_d| < \varepsilon_s \text{ or } \eta_d < \eta_s$$

(27)

If elasticities are measured on notional functions at the expected conflict points, then the side with the greater elasticity is that which attains the greater total utility. Hence, if it is known that multiplicative utility functions represent underlying agent behaviour, then to determine who attains the greater payoff utility, one need only measure price/income elasticities at the expected conflict prices on notional demand and supply curves and compare them.

These results follow from the fact that:

$$|\varepsilon_d| = \left| \frac{\partial \bar{Q}^d}{\partial \bar{P}^d} \right| \frac{\bar{P}^d}{\bar{Q}^d} = 1 + \frac{2\omega_d}{\bar{Q}}$$

$$\varepsilon_s = \left| \frac{\partial \bar{Q}^s}{\partial \bar{P}^s} \right| \frac{\bar{P}^s}{\bar{Q}^s} = \frac{\omega_s}{2\bar{Q}} - 1$$

It is easily shown that $|\varepsilon_d| = \varepsilon_s$ if $\bar{Q} = (\omega_d - \omega^d)/4$ or $\bar{Q} = Q^e/2$. 


V. Concluding Comments

This paper has attempted to outline a new theoretical approach to analyzing quotas. The relation of these new found results to conventional predictions should be discussed. Recall the two principal criticisms of traditional analyses: notional demand/supply theory is inapplicable and demanders may act collectively to improve their outcome utility. These two faults have been remedied by constructing an effective demand/supply theory relevant to quotas, and by enabling demanders to act collectively and bargain over a price for the quota level with sellers. In all cases some $P_d^*$ less than $P_d$ is predicted, thereby favouring buyers somewhat when compared to conventional analyses.

The analysis outlined may be considered more general, applying to non quota situations. The general Marshallian quantity-adjustment view of market operations (see Davies (1963) for the distinction between Marshallian and Walrasian market operations), which assumes agents formulate price demands/supplies for given quantities, fits neatly into the provided discussion, i.e. simply label $Q$ as the given quantity. Thus the foregoing discussion also seems applicable to any market which is better described by the "quantities given" label than the "prices given" label, and is hypothesized to trade out of notional equilibrium.

We conclude by pointing out some possible extensions to the analysis. First, the example provided in Section IV is a pure illustration showing that consistent reconciliation is in fact possible. These results are necessarily utility functional form specific and hence the notions concerning elasticities should not be generalised upon and taken out of context. Extensions to cover other utility functional forms (and flexible functional forms) seem warranted therefore. Second, the results are ZNH bargaining solution specific, hence extensions to incorporate uncertainty, learning, the role of time, etc. into the bargaining process should also be entertained, see for example Cross (1965).
References

Australian Tobacco Board. Annual Reports.


